Proceedings of the 40th IEEE
Conference on Decision and Control
Orlando, Florida USA, December 2001

Implementation of Tracking Controls for Constrained Discrete Time-Varying Systems via Receding Horizon Strategy

Ki Baek Kim†

Networking Laboratory, California Institute of Technology
1200 East California Boulevard, MS 256-80 Pasadena, CA 91125 USA
Center for Semiconductor Technology, Korea University
1, 5-ka, Anam-dong, Sungbuk-ku, Seoul 136-701, Korea
E-mail: kkb@cisl.snu.ac.kr

Abstract

In this paper, a receding horizon tracking control (RHTC) scheme is proposed for input and tracking-error constrained linear discrete time-varying systems. The control scheme is based on the minimization of a finite horizon cost function with finite terminal weighting matrices, which can easily be implemented by using linear matrix inequality (LMI) optimization. The proposed RHTC scheme is discussed in terms of the asymptotic property, feasibility, and closed-loop stability. It is shown that imposing the zero control weighting matrix can guarantee the asymptotic property and feasibility for some special systems and tracking commands, and the closed-loop stability for discrete-time systems.

1 Introduction

In control literature based on optimal control theory, there has been little investigation about how to implement a stabilizing tracking control for input/tracking-error constrained time-varying systems. The results in [1], [2] consider tracking control problems only for unconstrained systems. In order for the tracking control system to be a bounded-input and bounded-output one, the closed-loop stability should be guaranteed when the tracking command is zero. Although the results in [3], [4] consider general time-varying systems, they are not implementable for constrained time-varying systems.

In control literature based on optimal control theory, there have been only a few investigations into the asymptotic property for tracking control problems. The receding horizon tracking controls (RHTC) in [5], [1] show the asymptotic property with integrated control action only for unconstrained time-invariant systems and constant tracking commands. Thus, all those results in [5], [1] cannot show the asymptotic property when tracking commands are time-varying, or systems are input/tracking-error constrained or time-varying.

2 Receding Horizon Tracking Control

Consider a linear discrete time-varying system:

$$\begin{aligned} x(i+1) &= A(i)x(i) + B(i)u(i), \quad x(i_0) = x_0 \quad (1) \\ z(i) &= C(i)x(i) \end{aligned}$$

subject to input and tracking-error constraints:

$$\begin{cases} -u_{lim}(i) \le G_u(i)u(i) \le u_{lim}(i), \ i = 0, 1, \cdots, \infty \\ -\tilde{z}_{lim}(i) \le G_{\tilde{z}}(i)\tilde{z}(i) \le \tilde{z}_{lim}(i), \ i = 0, 1, \cdots, \infty. \end{cases}$$

where $z_r(i)$ the given tracking command, $\tilde{z}(i) = z(i) - z_r(i)$, $G_u(i) \in R^{l\times m}$, and $G_{\tilde{z}}(i) \in R^{r\times p}$.

For the system (1), consider the following constrained optimization problem at each time i:

$$J^* = \min_{u,\ Q_f(i)} \sum_{\tau=i}^{i+N-1} [\tilde{z}^T(\tau)Q(\tau)\tilde{z}(\tau) + u^T(\tau)R(\tau) u(\tau)] + \tilde{x}^T(i+N)Q_f(i)\tilde{x}(i+N) \quad (2)$$

subject to input and tracking-error constraints:

$$-u_{lim}(\tau) \le G_u(\tau)u(\tau) \le u_{lim}(\tau) \quad (3)$$

$$-\tilde{z}_{lim}(\tau) \le G_{\tilde{z}}(\tau)\tilde{z}(\tau) \le \tilde{z}_{lim}(\tau) \quad (4)$$

where $\tilde{x}(i) = x(i) - x_r(i)$, $x_r(i) = L(i)z_r(i)$, $R(\tau) = R^T(\tau) \ge 0$, and $Q(\tau) = Q^T(\tau)$ and $Q_f(i) = Q_f^T(i)$ are positive definite matrices.

$Q_f(i)$ is a critical parameter for the asymptotic property, feasibility, and closed-loop stability of the receding horizon tracking control (RHTC), so we introduce

$$Q_f(i) \geq C^T(\sigma)Q(\sigma)C(\sigma) + H^T(i)R(\sigma)H(i) + (A(\sigma) - B(\sigma)H(i))^T Q_f(i)(A(\sigma) - B(\sigma)H(i)). \quad (5)$$

Then, the first control $u^*(i)$ is called receding horizon tracking control (RHTC), which is obtained by solving the problem (2) subject to (3)-(5) for $\sigma = i + N$ every time i.

In literature, $R(\cdot)$ is always assumes to be positive definite. However, even if $R(\cdot) = 0$, there can exist an unique and bounded control input.

In the following, we investigate the asymptotic property, feasibility, and closed-loop stability of the proposed RHTC for constrained linear discrete time-varying systems. To this end, we introduce

$$R(\sigma)H_r(\sigma)\tilde{x}_r(\sigma) = 0,\ (I - B(\sigma)H_r(\sigma))\tilde{x}_r(\sigma) = 0 \quad (6)$$

where $\tilde{x}_r(i) = A(i)x_r(i) - x_r(i+1)$.

THEOREM 1 *Assume that the problem (2) subject to (3)-(5) and (6) for $\sigma = i+N$ is feasible at each time i where $u(\sigma) = -[H(i)\tilde{x}(\sigma) + H_r(\sigma)\tilde{x}_r(\sigma)]$ and $x(\sigma+1) = A(\sigma)x(\sigma) + B(\sigma)u(\sigma)$ satisfy (3) and (4). Then, the tracking-error of the system (1) with the resulting RHTC goes to zero.*

$R(\sigma) = 0$ is the easiest way to satisfy (6). However, the condition (6) is very conservative since it is satisfied only for some special systems and tracking commands.

Lemma 1 *At the time i, assume that for all $\sigma \geq i+N$, there exists $Q_f(i)$ satisfying (5), (6),*

$$\begin{bmatrix} \Gamma(\sigma) & G_u(\sigma)Y(i) \\ Y^T(i)G_u^T(\sigma) & S(i) \end{bmatrix} \geq 0, \quad \Gamma_{j,j}(\sigma) \leq \quad (7)$$

$$(u_{lim,j}(\sigma) - |(G_u(\sigma)H_r(\sigma)\tilde{x}_r(\sigma))_{j-th\ row}|)^2 \quad (8)$$

$$G_{\tilde{z}}(\sigma)C(\sigma)S(i)C^T(\sigma)G_{\tilde{z}}^T(\sigma) \leq E(\sigma)$$

$$E_{j,j}(\sigma) \leq \tilde{z}^2_{lim,j}(\sigma). \quad (9)$$

Then, $u(\sigma) = -[H(i)\tilde{x}(\sigma) + H_r(\sigma)\tilde{x}_r(\sigma)]$ and $\tilde{z}(\sigma)$ satisfy input and tracking-error constraints, respectively for all $\sigma \geq i+N$ if $\tilde{x}(i+N) \in \mathcal{E}_{Q_f(i)}$ where $\mathcal{E}_{Q_f(i)}$ is an ellipsoid defined as

$$\mathcal{E}_{Q_f(i)} = \{\xi \in R^n |\ \xi^T Q_f(i)\xi \leq 1\}. \quad (10)$$

THEOREM 2 *If the problem (2) subject to (3)-(5), (6), (8), (9) for all $\sigma \geq i+N$, and $x(i+N) \in \mathcal{E}_{Q_f(i)}$ is feasible at the initial time, then it is always feasible and thus, the resulting RHTC makes the tracking-error go to zero.*

From Theorem 2, we can mention the closed-loop stability for constrained linear time-varying systems when $z_r(\cdot) = 0$ as follows.

THEOREM 3 *If the problem (2) subject to (3)-(5), (8), (9) for all $\sigma \geq i+N$, and $x(i+N) \in \mathcal{E}_{Q_f(i)}$ with $z_r(\cdot) = 0$ is feasible at the initial time, then the closed-loop system with the resulting RHC is uniformly bounded and attractive. It is uniformly asymptotically stable if $C(\cdot)$ is positive definite.*

In the presentation, the proposed RHTC is illustrated via simulation examples.

3 Conclusion

The proposed RHTC scheme seems to be first trials to tackle the asymptotic property and feasibility of the tracking control for constrained discrete time-varying systems including time-invariant systems. It is a very important and useful observation that under the zero control weighting matrix, the closed-loop stability can be guaranteed for discrete-time systems. For this reason, the proposed RHTC scheme based on LMI optimization is expected to be applied to many tracking control problems especially when systems are constrained and/or time-varying.

Acknowledgement: This work was supported by the Brain Korea 21 Project.

References

[1] K. B. Kim, J. W. Lee, and W. H. Kwon, "Intervalwise receding horizon H_∞-tracking control for discrete linear periodic systems," *IEEE Trans. Automat. Contr.*, vol. 45, no. 4, pp. 747 – 751, 2000.

[2] A. Cohen and U. Shaked, "Linear discrete-time H_∞ optimal tracking with preview," *IEEE Trans. Automat. Contr.*, vol. 42, pp. 270–276, 1997.

[3] M. V. Kothare, V. Balakrishnan, and M. Morari, "Robust constrained model predictive control using linear matrix inequalities," *Automatica*, vol. 32, pp. 1361 – 1379, 1996.

[4] G. De Nicolao and L. Magni and R. Scattolini, "Stabilizing receding horizon control of nonlinear time-varying systems," *IEEE Trans. Automat. Contr.*, vol. 43, no. 7, pp. 1030 – 1036, 1998.

[5] W. H. Kwon and D. G. Byun, "Receding horizon tracking control as a predictive control and its stability properties," *Int. J. Control*, vol. 50, no. 5, pp. 1807–1824, 1989.